APPLICATION NOTE

Calculating Uncertainty Using Digital Multimeter Ratio Measurement Techniques

Introduction

Ratio measurements are used to compute the value and accuracy of an unknown input voltage relative to a known reference voltage. Ratio measurements can be easily performed using a modern digital multimeter (DMM) by measuring the unknown input and reference voltages and using their ratio to determine the exact value of the unknown input voltage.

This paper compares three DMM ratio measurement techniques for determining the traceable value and measurement uncertainty of an unknown input. It also demonstrates how ratio measurement techniques can be used to achieve traceable measurement uncertainties that approach an instrument's 24-hour stability or transfer accuracy specifications.

Overview

All DMM's perform a DC voltage measurement by comparing or ratio-ing the unknown voltage applied to its input terminals to the instrument's internal, calibrated voltage reference. The instrument's 90-day, 1-year, or 2-year accuracy specifications can be applied to the resulting measured value to determine its traceable uncertainty.

A "Ratio" measurement performs this same comparison but instead uses a second measurement of an external reference voltage that is generally known to a much greater precision than the DMM's 90-day or longer accuracy specification. By doing so, the DMM is used only as a short term stable transfer device and as such will contribute a much smaller error to the final measurement result.

If the DMM contributed zero errors, then the accuracy of the unknown voltage would be the same as the traceable uncertainty of the reference voltage used in the measurement. The DMM's error contribution will negatively impact the certainty of the final measurement; however, this error can be significantly less than simply measuring the unknown voltage and applying the appropriate 90-day (or longer) DMM accuracy specification.

Some DMM's incorporate an "automated" measurement of the unknown input voltage and the applied reference voltage and then automatically calculate and display the resulting voltage ratio value. This measurement configuration is shown in Figure 1.

The DMM will automatically measure the two applied voltages using internal switching and then automatically calculate the resulting voltage ratio value from the two independent measurements.

Any DMM can be used to manually perform these two measurements as shown in Figure 2. The user will then manually compute the voltage ratio value from these two individual measurements.

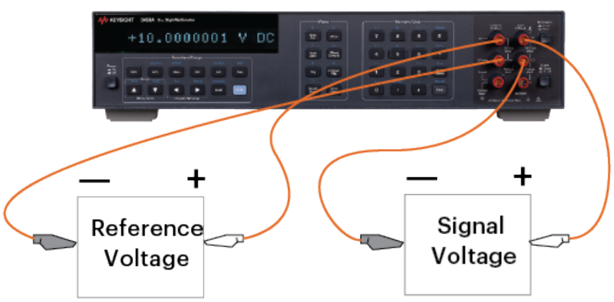

Figure 1. Automated measurements

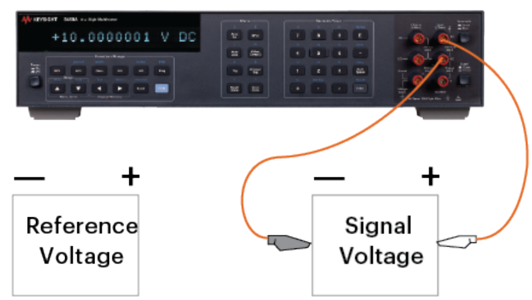

Figure 2. Manual measurements

Once the voltage ratio value is determined, the unknown value is calculated using Equation 1, independent of whether an “automated” or “manual” sequence of measurements were used to compute the voltage ratio value.

Equation 1: $V_{\text{UNKNOWN}} = V_{\text{REFERENCE}} \text{ x Ratio}$

The end purpose of using a ratio measurement is usually to determine the most precise, lowest uncertainty value of the unknown input voltage. The unknown voltage uncertainty, U, resulting from using any ratio measurement technique will always be described by the following relationship(s):

Equation 2a: $U_{\text{UNKNOWN}} = U_{\text{REFERENCE}} + U_{\text{RATIO DEVICE}}$ [Linear combination of errors]

or

Equation: 2b: $U_{\text{UNKNOWN}} = \sqrt{U^2_{\text{REFERENCE}} + U^2_{\text{RATIO DEVICE}}}$ [RSS combination of errors]

The “ratio device” error shown in Equation 2 above represents the total error contributed by the DMM in determining the “ratio” value used in Equation 1.

The final unknown input voltage and its traceable measurement uncertainty is expressed as:

Equation 3: $V_{\text{UNKNOWN}} \pm U_{\text{UNKNOWN}}$

The following ratio measurement examples and error calculations assume that a Keysight 3458A 8½ digit DMM is used. However, the ratio measurement techniques described can be applied to any DMM with the required measurement capability and specifications outlined in the examples below. The relevant 3458A specifications are reproduced in the Appendix at the end of this paper.

Example 1: Automatic Ratio Measurements

1:1 Ratio Measurement

$V_{REFERENCE}$	= 10.000020 V ± 1.5 µV or 10.000020 V ± 0.15 ppm	
$V_{UNKNOWN}$	= ?	
Ratio	= 1.0000025	*Displayed by DMM*
$V_{UNKNOWN}$	= 10.000020 V x 1.0000025 = 10.0000450 V ± $U_{UNKNOWN}$	

Uncertainty calculation (using the 3458A 24-hr DCV and ratio measurement specifications)

$U_{REFERENCE}$	= 0.15 ppm	
$U_{RATIO\ DEVICE}$	= $U_{DMM\ INPUT}$ + 1.5 x $U_{DMM\ REF\ INPUT}$	*Using Section 9, Ratio*
	= 0.55 ppm + 1.5 x 0.55 ppm	*accuracy specification*
	= 1.375 ppm	
$U_{UNKNOWN}$	= 0.15 ppm + 1.375 ppm	
	= 1.53 ppm of $V_{UNKNOWN}$	[Linear]
	= 1.38 ppm of $V_{UNKNOWN}$	[RSS]

Result:

$V_{UNKNOWN}$ = 10.0000450 V ± 15.3 µV or ± 1.53 ppm

10:1 Ratio Measurement

$V_{REFERENCE}$	= 10.000020 V ± 1.5 µV or 10.000020 V ± 0.15 ppm	
$V_{UNKNOWN}$	= ?	
Ratio	= 0.10000025	*Displayed by DMM*
$V_{UNKNOWN}$	= 10.000020 V x 0.10000025 = 1.00000450 V ± $U_{UNKNOWN}$	

Uncertainty calculation (using the 3458A 24-hr DCV and ratio measurement specifications)

$U_{REFERENCE}$	= 0.15 ppm	
$U_{RATIO\ DEVICE}$	= $U_{DMM\ INPUT}$ + 1.5 x $U_{DMM\ REF\ INPUT}$	*Using Section 9, Ratio*
	= 1.8 ppm + 1.5 x 0.55 ppm	*Accuracy specification*
	= 2.625 ppm	*and DMM 24-hr stability*
$U_{UNKNOWN}$	= 0.15 ppm + 2.625 ppm	*Specifications*
	= 2.78 ppm of $V_{UNKNOWN}$	[Linear]
	= 2.63 ppm of $V_{UNKNOWN}$	[RSS]

Result:

$V_{UNKNOWN}$ = 1.00000450 V ± 2.78 µV or ± 2.78 ppm

Example 2: Manual Ratio Measurements

1:1 Ratio Measurement

$V_{REFERENCE}$	= 10.000020 V ± 1.5 µV or 10.000020 V ± 0.15 ppm	*Known reference value*
$V_{REFERENCE}$	= 9.9999980 V	*Measured value*
$V_{UNKNOWN}$	= 10.000023 V	*Measured value*
Ratio	= 1.0000025	*Calculate ratio =* $V_{UNKNOWN}/V_{REFERENCE}$ *from the measured values.*
$V_{UNKNOWN}$	= 10.000020 V x 1.0000025 = 10.0000450 V ± $U_{UNKNOWN}$	

Uncertainty calculation (using the 3458A DMM 24-hr DCV specifications)

$U_{REFERENCE}$	= 0.15 ppm	
$U_{RATIO\ DEVICE}$	= $U_{DMM\ INPUT}$ + $U_{DMM\ REF\ INPUT}$	
	= 0.55 ppm + 0.55 ppm	
	= 1.10 ppm	
$U_{UNKNOWN}$	= 0.15 ppm + 1.10 ppm	
	= 1.25 ppm of $V_{UNKNOWN}$	[Linear]
	= 1.11 ppm of $V_{UNKNOWN}$	[RSS]

Result:

$V_{UNKNOWN}$ = 10.0000450 V ± 12.5 µV or ± 1.25 ppm

10:1 Ratio Measurement

$V_{REFERENCE}$	= 10.000020 V ± 1.5 µV or 10.000020 V ± 0.15 ppm	*Known reference value*
$V_{REFERENCE}$	= 9.9999980 V	*Measured value*
$V_{UNKNOWN}$	= 1.0000023 V	*Measured value*
Ratio	= 0.10000025	*Calculate ratio=* $V_{UNKNOWN}/V_{REFERENCE}$ *from the measured values.*
$V_{UNKNOWN}$	= 10.000020 V x 0.10000025 = 1.00000450 V ± $U_{UNKNOWN}$	

Uncertainty calculation (using the 3458A DMM 24-hr DCV specifications)

$U_{REFERENCE}$ = 0.15 ppm

	$V_{UNKNOWN}$ measured on DMM 1 V range		$V_{UNKNOWN}$ measured on DMM 10 V range
$U_{RATIO\ DEVICE}$	= $U_{DMM\ INPUT}$ + $U_{DMM\ REF\ INPUT}$		$U_{RATIO\ DEVICE}$ = $U_{DMM\ INPUT}$ + $U_{DMM\ REF\ INPUT}$
	= 1.8 ppm + 0.55 ppm		= 1.0 ppm + 0.55 ppm
	= 2.35 ppm		= 1.55 ppm
$U_{UNKNOWN}$	= 0.15 ppm + 2.35 ppm		$U_{UNKNOWN}$ = 0.15 ppm + 1.55 ppm
	= 2.50 ppm of $V_{UNKNOWN}$	[Linear]	= 1.70 ppm of $V_{UNKNOWN}$
	= 2.36 ppm of $V_{UNKNOWN}$	[RSS]	= 1.56 ppm of $V_{UNKNOWN}$

Result:

$V_{UNKNOWN}$ = 1.00000450 V ± 2.50 µV or ± 2.50 ppm measured on the 1 V range

$V_{UNKNOWN}$ = 1.00000450 V ± 1.70 µV or ± 1.70 ppm measured on the 10 V range

Example 3: Manual Ratio Measurements Using The DMM's Transfer Accuracy Specifications

1:1 Ratio Measurement

$V_{REFERENCE}$	= 10.000020 V ± 1.5 µV or 10.000020 V ± 0.15 ppm	Known reference value
$V_{REFERENCE}$	= 9.9999980 V	Measured value
$V_{UNKNOWN}$	= 10.000023 V	Measured value
Ratio	= 1.0000025	Calculate ratio = $V_{UNKNOWN}/ V_{REFERENCE}$ from the measured values
$V_{UNKNOWN}$	= 10.000020 V x 1.0000025 = 10.0000450 V ± $U_{UNKNOWN}$	

Uncertainty calculation (using 3458A DMM transfer accuracy specifications and conditions)

$U_{REFERENCE}$	= 0.15 ppm	
$U_{RATIO\ DEVICE}$	= $U_{DMM\ INPUT}$ + $U_{DMM\ REF\ INPUT}$	
	= 0.10 ppm + 0.10 ppm	
	= 0.20 ppm	
$U_{UNKNOWN}$	= 0.15 ppm + 0.20 ppm	
	= 0.35 ppm of $V_{UNKNOWN}$	[Linear]
	= 0.25 ppm of $V_{UNKNOWN}$	[RSS]

Result:

$V_{UNKNOWN}$ = 10.0000450 V ± 3.5 µV or ± 0.35 ppm

10:1 Ratio Measurement

$V_{REFERENCE}$	= 10.000020 V ± 1.5µV or 10.000020 V ± 0.15 ppm	*Known reference value*
$V_{REFERENCE}$	= 9.9999980 V	*Measured value*
$V_{UNKNOWN}$	= 1.0000023 V	*Measured value*
Ratio	= 0.10000025	*Calculate ratio= = $V_{UNKNOWN}/ V_{REFERENCE}$ from the measured values*
$V_{UNKNOWN}$	= 10.000020 V x 0.10000025 = 1.00000450 V ± $U_{UNKNOWN}$	

Uncertainty calculation (using 3458A transfer accuracy specifications and conditions)

$U_{REFERENCE}$	= 0.15 ppm	
$U_{RATIO\ DEVICE}$	= $U_{DMM\ INPUT}$ + $U_{DMM\ REF\ INPUT}$	NOTE: $V_{UNKNOWN}$ must be measured on the 10 V range
	= 0.55 ppm + 0.10 ppm	
	= 0.65 ppm	
$U_{UNKNOWN}$	= 0.15 ppm + 0.65 ppm	
	= 0.80 ppm of $V_{UNKNOWN}$	[Linear]
	= 0.67 ppm of $V_{UNKNOWN}$	[RSS]

Result:

$V_{UNKNOWN}$ = 1.00000450 V ± 0.80 µV or ± 0.80 ppm

Summary

The table below summarizes the ratio measurement results described in the examples above. The most precise ratio measurements are achieved when the unknown and reference voltages are similar as demonstrated in the 1:1 ratio measurement examples.

Measurement errors will increase for increasing ratio values – for example, a 10:1 ratio measurement will have greater uncertainty than a 1:1 ratio measurement. Similarly, measurement error will be minimized when the greatest care can be taken to eliminate instrument-contributed errors by manually performing the ratio measurements or by utilizing metrology-grade transfer measurement procedures as demonstrated in Example 3 where the 3458A's transfer accuracy specifications were used while observing all of their documented use conditions.

	10:1 Ratio Measurements (ppm uncertainty)				**1:1 Ratio Measurements (ppm uncertainty)**		
Uncertainty Calculation	Automatic	Manual		Manual Transfer	Automatic	Manual	Manual Transfer
		1 V Range	10 V Range				
Linear	2.78	2.50	1.70	0.80	1.53	1.25	0.35
RSS	2.63	2.36	1.56	0.67	1.38	1.11	0.25

Table 1. Summary of ratio measurement techniques.

Conclusion

This paper describes how to utilize a precision DMM to perform automated or manual ratio measurements to determine the value and measurement uncertainty of an unknown voltage compared to a known reference voltage. The paper also demonstrates how the resulting unknown voltage measurement uncertainty can be far less than the DMM's published 90-day, 1-year, or 2-year accuracy specifications by carefully applying these ratio measurement techniques.

Appendix

3458A DMM specifications are reproduced from its published data sheet (literature number 5965-4971E). The user may assume that these numbers represent k = 2, 95% confidence specifications.

Section 1: DC Voltage

Accuracy [ppm of reading (ppm of reading for Option 002) + ppm of range]

Range	24 hour	90 day	1 year	2 years
100 mV	2.5 + 3	5.0 (3.5) + 3	9 (5) + 3	14 (10) + 3
1 V	1.5 + 0.3	4.6 (3.1) + 0.3	8 (4) + 0.3	14 (10) + 0.3
10 V	0.5 + 0.05	4.1 (2.6) + 0.05	8 (4) + 0.05	14 (10) + 0.05
100 V	2.5 + 0.3	6.0 (4.5) + 0.3	10 (6) + 0.3	14 (10) + 0.3
1000 V	2.5 + 0.1	6.0 (4.5) + 0.1	10 (6) + 0.1	14 (10) + 0.1

Transfer accuracy/linearity

Range	10 min, T_{ref} ± 0.5 °C (ppm of reading + ppm of range)	Conditions
100 mV	0.5 + 0.5	• Following 4 hour warm-up. Full scale to 10% of full scale. • Measurements on the 1000 V range are within 5% of the initial measurement value and following measurement settling. • T_{ref} is the starting ambient temperature. • Measurements are made on a fixed range (> 4 min.) using accepted metrology practices.
1 V	0.3 + 0.1	
10 V	0.05 + 0.05	
100 V	0.5 + 0.1	
1000 V	1.5 + 0.05	

Section 2: Ratio (Automated)

Type Of Ratio	
DCV/DCV	Ratio = (input)/(reference)
ACV/DCV	Reference: (HI sense to LO) - (LO sense to LO)
ACDCV/DCV	Reference signal range: ± 12 V DC (autorange only)
Accuracy	
	± (input error + reference error) Input error = 1 x total error for input signal measurement function (DCV, ACV, ACDCV) Reference error = 1.5 x total error for the range of the reference DC input

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